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Experimental identification of longitudinal and vertical lift aerodynamic admittance functions of thin sections

Min Gao [1], Lei Yan [1,2,3], Xuhui He [1,2,3]

[1] *School of Civil Engineering, Central South University, Changsha, China, 410075*
[2] *National Engineering Research Center for High-Speed Railway Construction, Changsha, China, 410075*
[3] *Hunan Provincial Key Laboratory for Disaster Prevention and Mitigation of Rail Transit Engineering Structures, Changsha, China, 410075*

SUMMARY:
This study proposes a new separation method for the two-dimensional lift aerodynamic admittance functions, which can obtain the longitudinal and vertical lift aerodynamic admittance functions considering the longitudinal (*u*-) and vertical (*w*-) turbulence components. The method was first validated through an airfoil section, and then was extended to estimate the aerodynamic admittance of the streamlined bridge deck. To further verify the identification method, a quantitative comparison was made between the theoretical functions and the corresponding measured two-dimensional lift aerodynamic admittance functions obtained through wind tunnel tests on the streamlined box girder section.

Keywords: two-dimensional aerodynamic admittance, identification method, streamlined box girder

1. INTRODUCTION

The aerodynamic admittance function (AAF) relates turbulent fluctuations to lift on an airfoil and was proposed by Sears (1941). Since Davenport (1962) applied AAF to the field of bridge wind engineering, it has played a crucial role in analyzing buffeting response in large-span bridges. Due to limitations in wind field simulation techniques, it is challenging to match turbulence integral scales. Hence, identifying 2D AAF in turbulence may better predict bridge buffeting response. In addition, the accurate identification of the AAFs considering the *u* and *w* turbulence components is also a critical challenge for estimating buffeting response.This paper proposes a new method to identify *u* and *w* lift AAFs in turbulent flow. It is validated by comparing identified 2D AAF with theoretical solutions for an airfoil. The method is then extended to a streamlined box girder section.

2. MATHEMATICAL MODEL

Based on the measured spanwise coherence function and a general identification framework of two-wavenumber AAFs,this paper proposes a method to identify *u* and *w* 2D lift AAFs. Firstly, the Sears function and the two-wavenumber buffeting lift spectrum can be expressed as:

$$|\chi_{Sears}(k)|^2 = \left|\frac{J_0(k)K_1(ik)+iJ_1(k)K_0(ik)}{K_1(ik)+K_0(ik)}\right|^2 \tag{1}$$

$$S_L(k_1,k_2) = (\rho U b)^2\left[4C_L^2\left|\chi_{Lu}(\tilde{k}_1,\tilde{k}_2)\right|^2 S_u(k_1,k_2) + \left(C_L^{'}+C_D\right)^2\left|\chi_{Lw}(\tilde{k}_1,\tilde{k}_2)\right|^2 S_w(k_1,k_2)\right] \tag{2}$$

where k is the chordwise wavenumber; J_0, J_1 are Bessel functions of the first kind; K_0, K_1 are the modified Bessel function of the second kind; ρ is the air density; U is the mean wind velocity; b is the semi-width of the thin sections; C_L, C_D are the lift and drag coefficients, respectively; k_2 is the spanwise wavenumber; $S_L(k_1,k_2)$, $S_u(k_1,k_2)$ and $S_w(k_1,k_2)$ are the two-wavenumber spectra corresponding to the lift force, *u* and *w* components; $\left|\chi_{Lu}(\tilde{k}_1,\tilde{k}_2)\right|^2$ and $\left|\chi_{Lw}(\tilde{k}_1,\tilde{k}_2)\right|^2$ are the two-wavenumber AAFs of *u* and *w* turbulence.

Greenberg (1947) derived the theoretical solution of the 2D AAF for a thin airfoil in fluctuating wind based on potential flow theory, namely the Greenberg function. Horlock (1968) extended this theoretical solution and employed a method similar to the Sears analysis to derive the Horlock function. The expression for the Greenberg function and Horlock function, respectively, are:

$$\left|G(\tilde{k})\right|^2 = \left|1+\frac{1}{2}i\tilde{k}+C(\tilde{k})\right|^2, \left|H(\tilde{k})\right|^2 = \left|J_0(\tilde{k})+2iJ_1(\tilde{k})+C(\tilde{k})[J_0(\tilde{k})-iJ_1(\tilde{k})]\right|^2 \tag{3}$$

where $C(\tilde{k})$ is the Theodorsen function.

Thus, the theoretical solution ratio of *u* and *w* 2D AAFs can be obtained by Eq. (1) and Eq. (3).

$$\left|\lambda_G(\tilde{k})\right|^2 = \frac{\left|G(\tilde{k})\right|^2}{\left|\chi_{\mathrm{Sears}}(\tilde{k})\right|^2}, \left|\lambda_H(\tilde{k})\right|^2 = \frac{\left|H(\tilde{k})\right|^2}{\left|\chi_{\mathrm{Sears}}(\tilde{k})\right|^2} \tag{4}$$

Actually, it is difficult to accurately identify $\left|\chi_{Lu}(\tilde{k}_1,\tilde{k}_2)\right|^2$ and $\left|\chi_{Lw}(\tilde{k}_1,\tilde{k}_2)\right|^2$ through traditional wind tunnel tests without additional conditions. Hence, this paper assumes that the ratio between the *u* and *w* 2D AAFs of the airfoil section satisfies the ratio of the theoretical solution.

$$\frac{\left|\chi_{Lu}(\tilde{k}_1,0)\right|^2}{\left|\chi_{Lw}(\tilde{k}_1,0)\right|^2} = \frac{\left|G(\tilde{k})\right|^2}{\left|\chi_{Sears}(\tilde{k})\right|^2} = \left|\lambda_G(\tilde{k})\right|^2, \frac{\left|\chi_{Lu}(\tilde{k}_1,0)\right|^2}{\left|\chi_{Lw}(\tilde{k}_1,0)\right|^2} = \frac{\left|H(\tilde{k})\right|^2}{\left|\chi_{Sears}(\tilde{k})\right|^2} = \left|\lambda_H(\tilde{k})\right|^2 \tag{5}$$

Substituting Eq. (5) into Eq. (2) the separated 2D lift AAFs when $\tilde{k}_2$=0 an be obtained:

$$\left|\chi_{Lw}^G(\tilde{k}_1,0)\right|^2 = \frac{S_L(k_1,0)}{(\rho U b)^2\left[4C_L^2\left|\lambda_G(\tilde{k})\right|^2 S_u(k_1,0)+\left(C_L^{'}+C_D\right)^2 S_w(k_1,0)\right]}, \quad \left|\chi_{Lu}^G(\tilde{k}_1,0)\right|^2 = \left|\lambda_G(\tilde{k})\right|^2\left|\chi_{Lw}^G(\tilde{k}_1,0)\right|^2 \tag{6}$$

$$\left|\chi_{Lw}^H(\tilde{k}_1,0)\right|^2 = \frac{S_L(k_1,0)}{(\rho U b)^2\left[4C_L^2\left|\lambda_H(\tilde{k})\right|^2 S_u(k_1,0)+\left(C_L^{'}+C_D\right)^2 S_w(k_1,0)\right]}, \quad \left|\chi_{Lu}^H(\tilde{k}_1,0)\right|^2 = \left|\lambda_H(\tilde{k})\right|^2\left|\chi_{Lw}^H(\tilde{k}_1,0)\right|^2 \tag{7}$$

This paper will extend the $\left|\lambda_i(\tilde{k})\right|_{airfoil}^2$ to the streamlined box girder section. That is:

$$\left|\lambda_i(\tilde{k})\right|_{airfoil}^2 = \left|\lambda_i(\tilde{k})\right|_{streamlined\ box\ girder}^2 (i=G,H) \tag{8}$$

3. EXPERIMENT DESCRIPTION

The wind tunnel tests are carried out in the high-speed test section of the high-speed railway wind tunnel (CSU-1) at Central South University, China, using the same setup and box girder section models as described in Yan et al. (2023). For further details on the section models, spanwise spacings, instrumentation, grid setup, and characteristics of grid-generated turbulence, please refer to Yan et al. (2023). The airfoil and streamlined box girder sectional models with geometric ratio of 1:50 and the arrangement of the pressure taps are shown in Fig. 1.

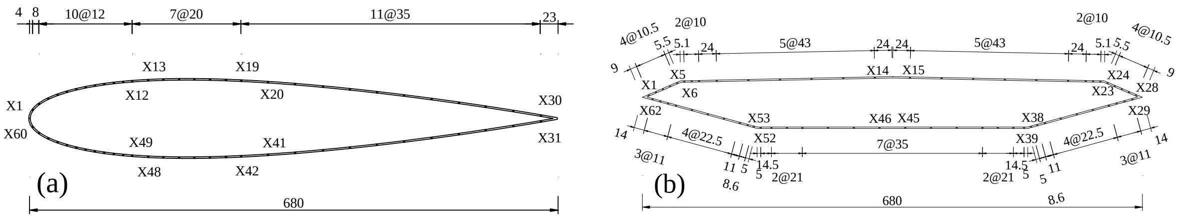

Figure 1. Schematic cross-sections of models and arrangement of pressure taps: (a) airfoil section, (b) streamlined box girder section. (Unit: mm, X + number represents the number of the pressure taps in the X cross-section)

4. RESULTS AND DISCUSSIONS

In Fig. 2, the proposed method successfully separates the longitudinal and vertical 2D lift AAFs in Grid B2. Comparing the results with the corresponding theoretical solutions, the separated 2D lift AAFs, which are based on $|\lambda_H(\tilde{k})|^2$, were found to be more reliable. This provides a theoretical foundation for extending the ratio relationship to bluff body sections and identifying the longitudinal and vertical 2D AAFs of the streamlined box girder section in this study.

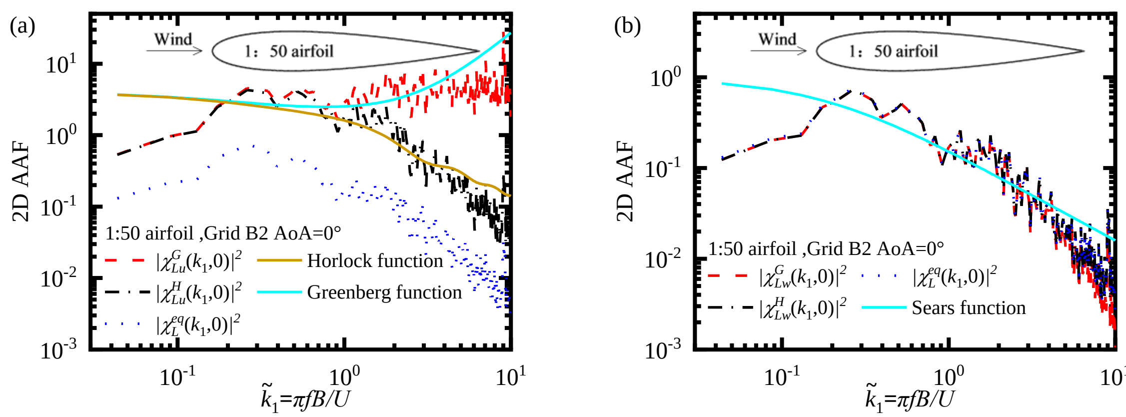

Figure 2. Measured results of 2D lift AAFs of 1:50 airfoil section: (a) Longitudinal, (b) Vertical.

Fig. 3 shows the measured results of the longitudinal and vertical 2D lift AAFs of the streamlined box girder section. It can be found that the results are close to the theoretical function by this method and decrease with the $\tilde{k}_1$ increasing, except in the low frequency range. However, due to the differences between the aerodynamic shape of the streamlined box girder and the airfoil, it is necessary to quantitatively discuss the results in Fig. 3 in order to validate the applicability of the method. Table 1 lists the integral area values of the $|\chi_L^{eq}(\tilde{k}_1,0)|^2$, $|\chi_{L_w}(\tilde{k}_1,0)|^2$ and the Sears function. It shows the close resemblance between the Sears function and the integral area value of $|\chi_{Lw}(\tilde{k}_1,0)|^2_{airfoil}$. The integral area value of $|\chi_{L_w}(\tilde{k}_1,0)|^2_{box\ girder}$ is slightly larger than the Sears function, which is consistent with previous studies and validates the applicability of the method.

By comparing the errors (ε) between the equivalent AAFs method and the proposed method, the results identified using the latter method show less error, thus illustrating its superiority.

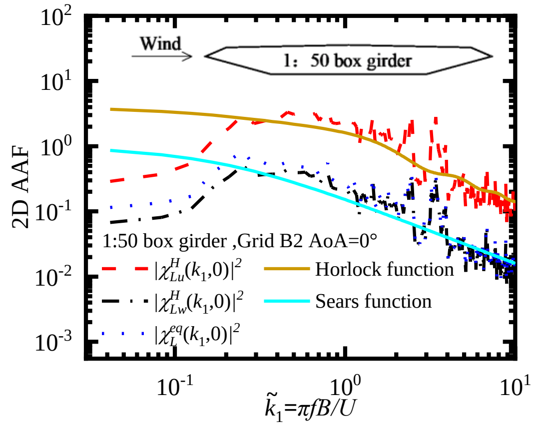

Figure 3. Measured results of 2D lift AAFs of 1:50 streamlined box girder section.

Table 1 Integral area values of the 2D lift AAFs ($\tilde{k}_1$=0.04~10)

2D lift AAF / cross-section	(A) Sears function	(B) $\left\|\chi_L^{eq}(\tilde{k}_1,0)\right\|^2$	(C) $\left\|\chi_{L_w}(\tilde{k}_1,0)\right\|^2$	ε_1=|A-C|	ε_2=|A-B|
1:50 airfoil	0.683	0.717	0.698	0.034	0.015
1:50 streamlined box girder		0.982	0.792	0.299	0.109

Note: The area values obtained above refer to the integral area of the 2D lift AAFs curve enclosed by its corresponding abscissa, ε represents the absolute error.

5. CONCLUSIONS

The theoretical solution ratio method is proposed for identifying 2D lift AAFs of an airfoil via the wind tunnel test in a turbulent wind field. A comparison between the theoretical functions and the measured results verified the correctness of AAFs identification. The method is extended to a streamlined box girder section, with good agreement between the theoretical solution and the corresponding 2D lift AAFs obtained by the wind tunnel test, verifying its feasibility and reliability.

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